An Investigation of Disjuncture between Graphing in School Mathematics and School Physics

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This paper reports an inquiry into approaches to graphing required in Physics and Applicable Mathematics tertiary entrance examinations (TEE), in Western Australia. The focus in the paper is on the different graphing practices in Physics and Applicable Mathematics in relation to gradient and transformed data, and in regard to use of graphics calculators. The differences mediate against transfer between the subjects and are one explanation for students' poor performance in graphing in the Physics TEE.

Introduction

> Candidates have always encountered difficulty with graphical questions (Examiners' report, 1995).
> Candidates have always had problems with graphical questions, and it was not any different this year (Examiners' report, 1998).

Examiners' comments like the above are common in relation to Physics Tertiary Entrance Examinations (TEE) in Western Australia and they prompted me to investigate if there was disjuncture between graphing practices in mathematics at the secondary level and those called on in the Physics TEE. By graphing practices, I mean actions associated with the production and interpretation of graphs. This paper reports selected findings of the inquiry, which is ongoing.

Assumptions of situated cognition theory underpin the analysis. Literature that I drew on included Cobb and Bowers (1999) critical comparison of various situated views and Wenger's (1998) formulation of learning within communities of practice. Literature on graphing as social practice (e.g., Roth & McGinn, 1997; Roth & Bowen, 2001) also informed the analysis.

Inquiry showed there is high call on gradient in solving Physics TEE questions. Gradient as 'continuous rate of change' and 'ratio' need to be brought to bear, where regularly (a) gradient is defined algebraically in terms of multiple parameters, (b) graphs involve transformed data and (c) values for calculating gradient need to be read off graphs. In secondary-school mathematics in Western Australia, the use of multiple parameters is uncommon and students have limited exposure to transforming data. Use of gradient, multiple parameters and the transformation of data are the foci of the paper.

The issue of students' apparent failure with graphing in the Physics TEE is significant, for graphical questions frequently attract substantial mark allocation. For instance in 2001, 12.5% of the total marks in the examination were allocated to drawing graphs or interpreting them. The investigation of graphing is relevant also to jurisdictions beyond Western Australia. Calls on slope are common in university entrance physics examinations, for instance, in the Victorian Certificate of Education, Advanced Placement in the United States and A-Levels in the United Kingdom; and data transformation appears in the A-Level examinations.

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Interpretative Framework

Definition of the Inquiry in Terms of Situated Views of Learning

From a situated viewpoint, inscriptions, including graphs, function as *boundary objects* (Roth & McGinn, 1998) in the discourses of people from different communities. As such, they can mediate *translation* of talk in mathematics classrooms and talk in physics classrooms. In discussing a graph in physics, students might realise that approaches to graphing in mathematics are relevant, and vice-versa. *Transfer* is another construct which is aligned with the translation. From a situated viewpoint, transfer requires that the social practices (in a broad sense) which are called upon in a problem situation be recognised from other task settings (Cobb & Bowers, 1999).

However, students do not necessarily translate or transfer practices between subjects. Furthermore, my initial premises in the investigation were that graphs are treated in different ways in school mathematics and school physics so that possibilities for translation and transfer are exceeded; and sometimes the different ways are inconsistent so that it is inevitable many students wallow in a sea of confusion.

Research Method

Cobb and Bowers (1999) distinguish that, from a situated viewpoint, the unit of analysis can be the individual in the social setting or group practices. The focus in the inquiry reported here is graphing practices that students, as a group, needed to bring to bear in the Physics TEE. I discerned these practices from working through the physics examinations for 1989-2001 and by comparing my solutions against the examiners' solutions. My over-riding task was to identify how changes in the TEE Physics syllabus in 1994 were reflected in the examinations, according to a range of question attributes. The analysis below relates to the 1994-2001 examinations and to graphing.

Thus, the analysis does not address approaches to graphing enacted by students but approaches that would have yielded them successful solutions. Furthermore, a key aspect of the inquiry was how the graphing approaches for the Physics TEE varied from those taught in mathematics and I founded the comparison on my recent twelve years experience of teaching upper-secondary mathematics in Western Australia.

Situated Perspectives on the Nature of Learning and Graphing

Wenger (1998) theorises learning and acting within *communities of practice* and highlights two key aspects: *participation* and *reification*. "Participation refers to a process of taking part" and "the term reification means 'making into a thing'" (p. 55-56). Students participate in graphing processes in an examination and some aspects of these processes are reified in the relationships that they recognise and the graphs they produce. Or, their processes of interpreting a given graph are, at least to some extent, reified in the explanations and numerical values, etc., which they write down. Moreover, participation produces reification and reification influences future participation.

Roth and McGinn (1997) provide alternative views on participation and the artefacts which result. In particular, they identify graphing practices and characterise graphs. They cast graphs as (a) semiotic objects, where the object and the event it re-presents are constructed by the observer and this involves adjustment and readjustment; or graphs might be (b) objects that don't signify events, for instance a plot of points with given co-ordinates. Graphs can also be (c) rhetorical devices, where they are used to present an argument, point to a relationship, support a claim. So, students might choose to provide a graph in an examination in support of an answer or as an answer, even if a graph is not required. Last graphs can be (d) conscription devices and enlist students in activity.

Citing Leinhardt, Zaslavsky and Stein, Roth and McGinn (1997) put forward the view that physical situations and algebraic rules are afforded different emphases, in relation to graphs, in mathematics and science. "Mathematics educators are, depending on the curricular topic, interested in algebraic rules, graphs, and the movement within and between these spaces. Most science educators, however, are more interested in the relationship between graphs and situations and . . . the relation between algebraic rules (such as motion equations, optical equations) and situations" (pp. 92-93).

Bowen and Roth (1998) further explore graphing in science and identify reasons for students' experiencing difficulty in learning to graph and with interpreting graphs, including lack of experience in a field and lack of specificity in teachers' narratives about graphs. The need for specificity arises because of students' lack of experience and, at least in science, "interpretations of a graph lie not in understanding the representation itself as a static object but rather in understanding the social actions through which a graph was originally constructed" (p. 86).

Roth and McGinn (1998), citing Latour, identify other attributes of graphs and inscriptions, in general. They can be moved, copied, rescaled, combined, translated (into other inscriptions) and incorporated in different contexts. Further, Roth and Bowen (2001) point to two major difficulties that students experience with graphs: "slope/height confusions and iconic interpretation" (p. 161). They give the instance of students' judging relative speed of two objects on the basis of the height on distance-time graphs instead of on the basis of the gradients of the graphs. They identify also that (a) because graphs have arbitrary relations to the things they represent, students need to know the conventions of graphing; and (b) situations which graphs represent are inherently under-determined by the information that is available on the graph.

Other Situated Views and Their Relevance to the Inquiry

Nespor (1994) articulates a 'big picture' of networks of communities of practice and describes how people join communities, are mobilised along trajectories of practice within them, then move to other parts of the network. The context of the analysis was the passage of physics students through an undergraduate course during which they assumed the routines of physicists. Parallels can be drawn between the circumstances of the undergraduates and the Physics TEE students. The TEE is a rite of passage to university. Students must comply with the graphing practices expected by the examiners. So, views of the 'university community of physicists' as to what are appropriate practices in graphing underpin the practices called on in the Physics TEE: the examination panel comprises two university representatives and one teacher. Furthermore, past examination papers and examiners' solutions provide the best guide to what might be required, for the syllabus makes only brief mention of graphing. It specifies: "students should be able to . . . present and interpret experimental data in graphical and tabular form" (Curriculum Council, 2000, p. 125); "practical work is intended to . . . promote the development of practical skills, including . . . recording, analysis and graphing of data" (p. 128). As well, four types of graphs are listed: displacement/time and displacement/distance (or position) for progressive waves; displacement-distance graphs to illustrate standing (stationary) waves; and the stress/strain curves for typical brittle and ductile materials.

Another facet of students' graphing is the technologies that they have at their disposal and these, too, originate in sources outside school physics. Technologies, including tools, change how people go about an activity (Wenger, 1998). The tools Physics TEE students have which structure their graphing include, since 1998, non-CAS (computer algebra system) graphics calculator. Examiners' responses to the use of the calculators in the Physics TEE are quoted in the analysis.

Analysis

A question taken from the year 2001 Physics TEE (Curriculum Council, 2001) illustrates the call on gradient and the use of transformed data:

> Geraldine was investigating the speed of waves along stretch strings. She generated these waves by plucking a 0.760 m length of guitar string. She knew the speed was given by
>
> $$v = \sqrt{\frac{T}{\mu}}$$
>
> where T is the tension in the string and μ is the mass per unit length. She plotted her results in the graph as shown [see Figure 1].

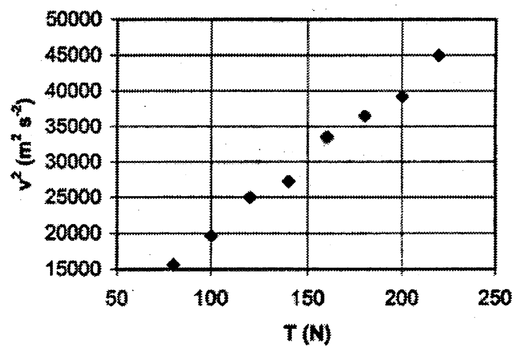

Figure 1. Graph included in the examination question.

> (a) (i) Why did Geraldine plot v^2 against T and not just v against T?
> (ii) What are the units of μ.
>
> (b) Use the graph to determine the best experimental value of μ for this string. Show your working clearly.
>
> Parts (c) and (d) followed. They did not require use of the graph so are not included.

The official solutions in the examiners' report showed the following:

> (a) (i) This makes the graph a straight line. (ii) μ = ... the units of μ are kg m^{-1}
>
> (b) Slope = (45000 – 15000) ÷ (210 – 75)
> Since $v^2 = (1/\mu)T$, slope = $1/\mu$ = 3000/135, from which μ = 4.50 g m^{-1}. [3000 is in error and should be 30000, giving μ = 4.50 x 10 g m^{-1}]

In each of the years 1994-2001 there has been a question in the Physics TEE of this type--asking for calculations based on a graph of transformed data. Topics and specification vary. For instance, besides the text of the question, in 1994 the equation $s = X + \{\tau / 2rm\} t^2$ was given, data for *s* (distance) and *t* (time) were tabulated and the parameters τ , *r* and *m* were defined. Students were asked to use the data to draw a straight line, calculate τ from the gradient, and calculate X, the y intercept. In 1995 the topic was sound waves with frequency f in a tube length l, the length to produce a standing wave. The equation $l = (v/4)(1/f) - e$ was given and the parameters v and e were defined. Students were to transform l, f data given in a table, in order to plot a straight line; then, determine the parameters v and e using the gradient and y intercept.

Transformations Required and Comparison with Transformations in Mathematics

The examples illustrate that in some questions data was already transformed and plotted (the 2001 question), and in other questions students needed to decide what transformation was required (see the 1994 and 1995 questions). Two types of transformation have been requested in the period 1994-2001: squaring in 1994, 1996, 1997 and 2001, and taking the reciprocal in 1995 and 1999.

In Applicable Mathematics (in Western Australia), which the majority of Physics TEE students study, students are expected to be able to transform data in exponential relationship into log-linear form and to plot and interpret log-linear graphs (Curriculum Council, 2000). As well, they would encounter adding a constant to co-ordinate values and multiplying co-ordinate values by a constant as an introduction to the effects of 'change of origin' and 'change of scale' on summary statistics. Transformations of function equations, $y = f(x)$ to $y = 2\,f(x)$ etc., and function graphs are included, but the processes bear little resemblance to those required in the above physics questions. They constitute movement within and between the algebraic and graphical spaces that Roth and McGinn (1997) identify, whereas in the above examples movement was between the physical situation and numerical, graphical and algebraic objectification of it.

So, in regard to the transformation of data in the above type of physics questions, there is minimal overlap with transformations in Applicable Mathematics. There was opportunity for overlap until 1993 with the log-linear transformation of exponential relationships, which relate to nuclear decay in physics. But questions about radioactive decay, at least from 1989-1993, relied on raw, untransformed data. Radioactive decay is not now in the Physics TEE course.

Line of Best Fit

Instructions and the official solutions for the Physics TEE indicate that students are expected to hand draw lines of best fit. As well, fitting the line by eye seems expected because marks are not allocated for specific points on the line and because 'latitude' is allowed in values students read from the graph (e.g., Examiners' report, 1999).

In Applicable Mathematics, lines of best fit or least squares regression lines might be introduced through hand methods and mean values are used to position the line. But, in my experience, from 1994-1997 determination of linear regression lines would be passed to a scientific calculator and, since 1998, to a graphics calculator. Both types of calculator produce the gradient and y intercept and, if a hand drawn graph is needed, it is drawn using these values or by retrieving points on the line from the calculators.

Moreover, students are expected to use their calculators in the Applicable Mathematics TEE to calculate the line before plotting it by hand (e.g., in 1997). Sometimes the graphical form of the line is not required and only the algebraic form is called for (e.g., in 1998). The graph produced on a graphics calculator can be relied on, for example, to identify outliers.

Students are expected, as well, to use their calculators for exponential regression, with raw (untransformed) data (e.g., in 1999).

Only one examiners' report in physics for the period 1994-2001 has acknowledged students could use their calculators for lines of best fit or, more particularly, use them to obtain the x intercept and gradient of a line after it had been drawn: "The wording of the question included an 'otherwise' to specifically permit the use of a statistical calculator to obtain both the intercept and gradient of the line of regression. Unfortunately this was frequently interpreted as pick any two points to obtain the gradient, throwing away the benefits of drawing a best straight line in the first place" (Examiners' report, 1996). However, the wording "From the graph **or otherwise** find . . ." (bolding included) didn't give any strong hint to use the technology; and students might not have recognised to use their calculators because the format of previous examination questions didn't invite the use. In another instance, an examiner's comment suggested he/she didn't realise the technology might be used for gradient: "Many did not show how they found the gradient in part b) but obtained the answer anyway" (Examiners' report, 2000).

So, it is not necessarily the case that the presence of a technology alters significantly how people go about an activity. Moreover, changes in assessment are fundamental to how students adopt new technologies in a subject (Berger, 1998). Following this line of argument, because the responses to the use of calculators in mathematics and physics have differed in tertiary entrance examinations, it is likely that classroom graphing practices are different, at least for the line of best fit. Also, the terminology 'line of best fit' predominates in the Physics TEE and 'least squares regression line' predominates in the Applicable Mathematics TEE; and the object associated with the 'line' in physics is likely to be a graph, while the reification of 'line' in mathematics is frequently an algebraic equation. All these factors contribute to disjuncture between the two subjects, to the possible detriment of learning, for connections between subject domains are known to contribute to learning being meaningful (Hiebert & Carpenter, 1992).

Furthermore, data and graphs are commonly transformed by scientists in the field (e.g., Roth, Hawryshyn & Haimberger, 2001) and this is only practical via the use of computer-technologies. On the other hand, some dispute that student understanding of graphs and graphing is enhanced by the use of computers (Berg & Smith, 1994).

Gradient

Having hand-drawn the line of best fit, solutions in the official TEE reports indicate that physics students are to read the co-ordinates of two points on the line and calculate gradient using rise/run. This approach is also widely used in mathematics, although with data analysis at upper-secondary level the gradient is commonly retrieved from a calculator, possibly leading to deskilling in regards the hand method. The y intercept is also retrieved from the calculator. In addition, the graphics calculator provides the equation for the line so that students don't need to formulate it.

Furthermore, calculations which follow finding the gradient differ in the two subjects. Physics TEE questions usually ask for values for parameters that make up the gradient (e.g., value of τ where the gradient was $\tau/(2rm)$, in 1994). In Applicable Mathematics, once the gradient and intercept have been determined, typically questions ask students to predict values for the dependent variable for given values of the independent variable (e.g., predict s given t, in relation to the 1994 question); and, in the instance of log-linear equations, the task is to change them to exponential form.

Examiners' comments indicate that, in physics, deducing values of parameters from the gradient is a point at which difficulties occur. For instance: "Candidates . . . are still not comfortable with calculating quantities from the gradients of graphs" (Examiners' report, 1999). The parameters often are associated with properties of the physical setup. However, the straight-lines produced by transformed data bear minimal if any physical resemblance to the setup. In other words, they grossly under-determine it. So, in 1994, the graph of distance s against t^2 did not indicate visually that the rate of change of distance increased over time, and did not signify visually anything about τ, the torque applied to the wheels of a car, which students were asked to determine.

One purpose of manipulating artefacts of an activity is to reveal features of the concrete situation (Hershkowitz, Schwarz & Dreyfus, 2001; Roth et al., 2001) but this regularly has not been the case with lines of fit in the Physics TEE questions. Instead of features on the graphs being able to be linked clearly with characteristics of a phenomenon, plotting transformed data has been a technology for producing straight lines, leading to the computation of gradient and y intercept. The abstract nature of the exercise explains students' difficulties.

Multiple parameters arguably also exacerbate the difficulties: students' mean scores were 33% for the1994 question (5 parameters): 44% in 1995 (2 parameters) and 55% in 2001 (1 parameter). Another aspect is the variables are not consistently shown in the same font (regular or italics) in the questions and neither are the parameters (e.g., v and e, in 1995), which could be another source of confusion for students. Another inconsistency is the way the y intercept is treated. In 2001 (see above) the equation of the line of best fit matching the gradient given in the solutions was $\mathrm{v}^2 = (30000/135)\mathrm{T} - 5000/3$. So, in saying the relationship was $\mathrm{v}^2 = (1/\mu)\mathrm{T}$, the y intercept '-5000/3' was ignored. In contrast, in the 1995 question the y intercept had significance in relation to the physical situation that the line of best fit was representing. How to treat the constant term is potentially another source of confusion for students.

Concluding Discussion

In regard to graphing in the Physics TEE, the issues of transformation of data, line of best fit and gradient have been touched upon. The practice of transforming data is encountered in Applicable Mathematics, but there has been no intersection between the specific transformations that are included in the syllabus for that subject and the ones encountered in the Physics TEE for the period 1994-2001. Moreover, the terminology 'line of best', the processes of the production of the line, its reified form and actions upon its gradient have been identified as differing from those in Applicable Mathematics.

The differences limit the scope for transfer of graphing practices from mathematics into solving physics problems of the type that have been discussed. One reason for the differences is utilisation of scientific calculators then graphics calculators in Applicable Mathematics and minimal recognition of them for fitting lines by examiners of the Physics TEE. So, the graph, rather than being a boundary object and site for translation of practices between school mathematics and school physics, stands as an object marking disjuncture between the subjects.

The analysis highlights that inclusion of technologies can compound lack of connection between subjects and might burden students in that they are called on to produce hand methods in one subject and technology-based methods in another. An implication is the need for dialogue between communities in the network of communities participating in education, in particular, in relation to practices expected of students as they attempt the rite of passage from one community to another and when technologies are introduced.

There is not room in this short paper to discuss to any great extent other aspects of graphing in the Physics TEE. However, there are domains of overlap with graphing in mathematics. For instance, in the year 2000 Physics TEE students were asked to estimate acceleration and distance from a velocity-time graph. Most Physics TEE students would have encountered drawing a tangent to the curve and estimating area under it in Year 11 Introductory Calculus. The mean score for the population was 58%. So, it is not that students will be able to graph in physics if they can transfer practices from mathematics, but that transfer between subjects is desirable, and the significance of the inquiry in this paper is identification of an area where more overlap is possible.

Acknowledgment

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References

Berg, C. A., & Smith, P. (1994). Assessing students' abilities to construct and interpret line graphs: Disparities between multiple-choice and free-response instruments. *Science Education, 78*, 527-554.

Berger, M. (1998). Graphics calculators: An interpretative framework. *For the Learning of Mathematics, 18*(2), 13-20.

Bowen, M. J., & Roth, W. -M. (1998). Lecturing graphing: What features of lectures contribute to student difficulties in learning to interpret graphs? *Research in Science Education, 28*(1), 77-90.

Cobb, P., & Bowers, J. (1999). Cognitive and situated learning perspectives in theory and practice. *Educational Researcher, 28*(2), 4-15.

Curriculum Council (2000). *2000/2001 Syllabus Manual Year 11 and Year 12 subjects.* Perth, WA: Curriculum Council.

Curriculum Council (2001). *Physics tertiary entrance examination.* Perth, WA: Curriculum Council.

Hiebert J., & Carpenter, T. P. (1992). Learning and teaching with understanding. In D. A. Grouws (Ed.), *Handbook of research in mathematics teaching and learning* (pp. 65-97). New York, NY: McMillan

Hershowitz, R., Schwarz, B. B., & Dreyfus, T. (2001). Abstraction in context: Epistemic actions. *Journal for Research in Mathematics Education, 32*(2), 195-222.

Nespor, J. (1994). *Knowledge in motion.* Washington, DC: Falmer.

Roth, W. -M., & Bowen G. M. (2001). Professionals read graphs: A semiotic analysis. *Journal for Research in Mathematics Education, 32*(2), 59-194.

Roth, W. -M., & McGinn M. G. (1997). Graphing: A cognitive ability or cultural practice. *Science Education, 81,* 91-106.

Roth, W. -M., & McGinn M. G. (1998). Inscriptions: Towards a theory of representing as social practice. *Review of Educational Research, 68*(1), 35-59.

Roth, W. -M., Hawryshyn, C., & Haimberger, T. (2001, March). *Graphing as emergent competence: A cultural-historical approach.* Paper presented at the annual conference of the National Association for Research in Science Teaching, St Louis, MO.

Wenger, E. (1998). *Communities of practice: Learning meaning, and identity.* Cambridge: Cambridge University.